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Solutions of thermal problems in friction welding—comparative study

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Abstract—Solutions of thermal problems in friction welding are discussed. Both analytical solutions of the heat flow equations for the problem analysed as well as numerical and experimental studies are given. A comparison of various approaches for the analysis of thermal effects in friction welding provides valuable data as to the accuracy of the methods.

INTRODUCTION

In this paper the heat flow problems in friction welding will be considered. Friction welding is a process in which the heat for welding is produced by direct conversion of mechanical energy to thermal energy at the interface of the workpieces, without the application of electrical energy, or heat from other sources, to the workpieces. Friction welds are made by holding a non-rotating workpiece in contact with a rotating workpiece under constant or gradually increasing pressure until the interface reaches welding temperature, and then stopping rotation to complete the weld.

The frictional heat developed at the interface rapidly raises the temperature of the workpieces, over a very short axial distance, to a value approaching, but below, the melting range; welding occurs under the influence of a pressure that is applied while the heated zone is in the plastic temperature range. Friction welding is classified as a solid-state welding process in which joining occurs at a temperature below the melting point of the work metal. If incipient melting does occur, there is no evidence in the finished weld because the metal is worked during the welding stage. This paper describes some methods for the analysis of the transient temperature distribution in the vicinity of the weld for arbitrary axisymmetric rods. The common assumption in attempting to provide an analytical solution to such a problem is the postulated temperature independence of all material properties. No such simplifications have to be done in a numerical approach. The finite element method used in its incremental form makes it possible to account for arbitrary variations of all material characteristics during the process. Experimental verifications of the methods [1] provide interesting data as to the accuracy of the methods.

EQUATIONS AND THEIR SOLUTIONS

Analytical solutions

Consider a friction welding of two rods of diameter $2r_0$. The rate of heat generated in the process of friction welding at the place of abutment will be denoted by q. This value is usually a function of time t, and can be expressed in the form $q(t) = q_0 \exp mt$, where m is the parameter of the process. The magnitude $q_2 = q/\pi r_0^2$, is introduced, which characterizes the heat rate provided at the place of contact of the elements. The heat flow equation can be written in the form [2–7]

$$\frac{\partial T}{\partial t} = q\frac{\partial^2 T}{\partial x^2} - bT + \frac{q_0}{c\rho}\exp mt\delta(x) \tag{1}$$

where T is the temperature, x the coordinate, $\delta(x)$ the Dirac function, $q = k/c\rho$, $b = 2h/c\rho r_0$, k the thermal conductivity, c the heat capacity, ρ the density and h the surface film conductance.

The boundary conditions are expressed as

$$\frac{\partial}{\partial x}T(\pm\infty, t) = 0, \quad T(x, 0) = 0. \tag{2}$$

Using a Laplace transformation [2, 8]

$$\bar{T}(x, s) = \int_0^\infty T(x, t)\,e^{-st}\,dt \tag{3}$$

we get

$$a\bar{T}''(x, s) - (b+s)\bar{T}(x, s) + \frac{q_0}{c\rho(s+m)} = 0,$$
$$\bar{T}'(\pm\infty, s) = 0. \tag{4}$$

Since

$$\bar{T}(-x, s) = \bar{T}(x, s)$$

we have

$$\bar{T}(x, s) = \frac{q_0\sqrt{a}}{2kF(s-m)\sqrt{(s+b)}} \times \exp\left(-\sqrt{(s+b)}\cdot\frac{x}{\sqrt{a}}\right), \quad x \geqslant 0. \tag{5}$$

Hence we obtain

$$T(x,t) = \frac{q_0\sqrt{a}}{2kF}\cdot\frac{\exp mt}{2\sqrt{(b+m)}}\left\{\exp\left[-\sqrt{\left(\frac{b+m}{a}\right)}\cdot x\right]\right.$$
$$\times\operatorname{erfc}\left[\frac{x}{2\sqrt{(at)}}-\sqrt{((b+m)t)}\right]$$
$$\left.-\exp\left[\sqrt{\left(\frac{b+m}{a}\right)}\cdot x\right]\operatorname{erfc}\left[\frac{x}{2\sqrt{(at)}}+\sqrt{((b+m)t)}\right]\right\}. \quad (6)$$

Consider equation (6) with dependence on the sign $(b+m)$ $b+m > 0$. Introduce the new function

$$\psi_1(\rho_1,\tau) = \tfrac{1}{2}\operatorname{erfc}\left(\frac{\rho_1}{2\sqrt{\tau}}-\sqrt{\tau}\right)$$
$$-\tfrac{1}{2}\exp^2\rho_1\operatorname{erfc}\cdot\left(\frac{\rho_1}{2\sqrt{\tau}}+\sqrt{\tau}\right), \quad (7)$$

where

$$\rho_1 = \sqrt{\left(\frac{b+m}{a}\right)}x, \quad \tau = (m+b)t.$$

Then we get, from equation (6),

$$T(x,t) = \frac{q_0}{2kF}\sqrt{\left(\frac{a}{b+m}\right)}\exp\left[mt-x\sqrt{\left(\frac{b+m}{a}\right)}\right]$$
$$\times\psi_1\left[\sqrt{\left(\frac{b+m}{a}\right)}x,(b+m)t\right]. \quad (8)$$

For $m = 0$, $b > 0$ and then equation (8) transforms to

$$T(x,t) = \frac{q_0}{2kF}\sqrt{\left(\frac{a}{b}\right)}\exp\left(-x\sqrt{\left(\frac{b}{a}\right)}\right)$$
$$\times\psi_1\left(\sqrt{\left(\frac{b}{a}\right)}x,bt\right); \quad x\geqslant 0. \quad (9)$$

For small values of the x coordinate, equation (6) can be expressed in the following form, with sufficient accuracy:

$$T(x,t) \cong \frac{q_0}{2kF}\exp mt\left[\sqrt{\left(\frac{a}{b+m}\right)}\operatorname{erf}\sqrt{((b+m)t)}-x\right];$$
$$x\geqslant 0. \quad (10)$$

Consider the case $(b+m) = 0$. Then equation (6) has the form

$$T(x,t) = \frac{q_0}{kF}\exp(-bt)\sqrt{(at)}$$
$$\times\operatorname{ierfc}\frac{x}{2\sqrt{(at)}}; \quad x\geqslant 0, \quad (11)$$

where

$$\operatorname{ierfc}u = \int_u^\infty \operatorname{erfc}\,du. \quad (12)$$

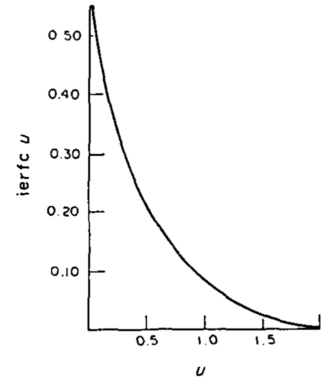

FIG. 1. Diagram of function ierf c.

The diagram of function ierfc is given in Fig. 1. At the place of contact, assuming $bt \ll 1$, the temperature is given by

$$T(0,t) = \frac{q_0}{\sqrt{(\pi kF)}}\exp(-bt)\sqrt{(at)}. \quad (13)$$

If $m = b = 0$ and $q_0 = \text{const}$, then the temperature in the rods at the place of abutment can be described by the equation

$$T(0,t) = \frac{q_0\sqrt{t}}{\sqrt{(\pi kc\rho F)}}. \quad (14)$$

The case $b+m < 0$ appears in the process of cooling. Denote the time of heating by t_c. For $t \geqslant t_c$ we get

$$T(x,t) = \frac{q_0}{2kF}\sqrt{\left(\frac{a}{b+m}\right)}\exp\left(mt-x\sqrt{\left(\frac{b+m}{a}\right)}\right)$$
$$\times\left\{\psi_1\left[\sqrt{\left(\frac{b+m}{a}\right)}x;(b+m)t\right]\right.$$
$$\left.-\psi_1\left[\sqrt{\left(\frac{b+m}{a}\right)}x;(b+m)(t-t_c)\right]\right\}. \quad (15)$$

For $x = 0$, the temperature is given by

$$T(0,t) = \frac{q_0}{2kF}\sqrt{\left(\frac{a}{b+m}\right)}$$
$$\times\exp mt[\operatorname{erf}\sqrt{((b+m)t)}-\operatorname{erf}\sqrt{((b+m)(t-t_c))}]. \quad (16)$$

Consider the variation of temperature in rods along the radius. We assume $T = T(r,x,t)$, where (r,x) is the cylindrical coordinate system. The heat flow equation in this case can be written as

$$c\rho\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial x^2}\right)$$
$$+[q_2(r)-q_2]\delta(x). \quad (17)$$

The boundary conditions have the form

$$\frac{\partial}{\partial x}T(r, \pm\infty, t) = 0, \quad \frac{\partial}{\partial r}T(r_0, x, t) = 0,$$

$$\frac{\partial}{\partial r}T(0, x, t) = 0, \qquad T(r, x, 0) = 0. \quad (18)$$

Introduce the following non-dimensional magnitudes

$$\theta = \frac{kT}{r_0 q_2}, \quad (19)$$

$$\tau = \frac{at}{r_0^2}, \quad (20)$$

$$\rho = r/r_0, \quad \xi = x/r_0, \quad (21)$$

$$f(r) = \frac{q_2(r)}{q_2} - 1. \quad (22)$$

The equation determining non-dimensional temperature $\theta(\rho, \xi, \tau)$ has the form [2, 8]

$$\theta(\rho, \xi, \tau) = \int_0^\tau \frac{d\tau'}{\sqrt{(4\pi(\tau-\tau'))}} \exp\left[-\frac{\xi^2}{4(\tau-\tau')}\right] \times 2\sum_{n=0}^{\infty} \exp[-\mu_n^2(\tau-\tau')] \frac{F(\mu_n)}{J_0^2(\mu_n)} J_0(\mu_n\rho), \quad (23)$$

where the heat source is given as $f(\rho) = \delta(\tau-\tau')$, τ' is the heating time, μ_n are solutions of the equation $J'_0(\mu_n) = J_1(\mu_n) = 0$, $n = 1, 2, \ldots$; $F(\mu_n) = \int_0^1 f(\rho) J_0(\mu_n\rho)\rho \, d\rho$; J_0, J_1 are Bessel functions.

Equation (23) can be written as

$$\theta(\rho, \xi, \tau) = \sum_{n=0}^{\infty} \frac{F(\mu_n)}{J_0^2(\mu_n)} J_0(\mu_n\rho) \times \int_0^\tau \frac{d\tau}{\sqrt{(\pi t)}} \exp\left(-\mu_n^2\tau - \frac{\xi^2}{4\tau}\right) \quad (24)$$

or introducing a function $\psi_1(\rho, \tau)$

$$\theta(\rho, \xi, \tau) = \sum_{n=0}^{\infty} \frac{F(\mu_n)}{\mu_n J_0^2(\mu_n)} J_0(\mu_n\rho) \times \exp(-\mu_n\rho)\psi_1(\mu_n\xi; \mu_n^2\tau). \quad (25)$$

For $\xi = 0$, we have

$$\theta(\rho, 0, \tau) = \sum_{n=0}^{\infty} \frac{F(\mu_n)}{\mu_n J_0^2(\mu_n)} \cdot J_0(\mu_n\rho) \operatorname{erf} \mu_n\sqrt{\tau}. \quad (26)$$

For $\rho = 0$ and $\rho = 1$

$$\theta(0, 0, \tau) = \sum_{n=0}^{\infty} \frac{F(\mu_n)}{\mu_n J_0^2(\mu_n)} \operatorname{erf} \mu_n\sqrt{\tau}, \quad (27)$$

$$\theta(1, 0, \tau) = \sum_{n=0}^{\infty} \frac{F(\mu_n)}{\mu_n J_0(\mu_n)} \operatorname{erf} \mu_n\sqrt{\tau}. \quad (28)$$

In order to get a solution for $\theta(\rho, \xi, \tau)$, it is necessary to define q_2 with a dependence on radius r.

Assume

$$q_2(\rho) = q_2 \sum_{m=1}^{\infty} \varepsilon_m \rho^m. \quad (29)$$

Then we can write

$$F(\mu_n) = \int_0^1 \left[\sum_{m=1}^{\infty} \varepsilon_m \rho^m - 1\right] J_0(\mu\rho)\rho \, d\rho = \sum_{m=1}^{\infty} \varepsilon_m \int_0^1 J_0(\mu\rho)\rho^{m+1} \, d\rho = \sum_{m=1}^{\infty} \varepsilon_m F_m(\mu), \quad (30)$$

where

$$F_m(\mu) = \int_0^1 J_0(\mu\rho)\rho^{m+1} \, d\rho = \frac{1}{\mu} J_1(\mu) + \frac{m}{\mu^2} J_0(\mu) - \frac{m^2}{\mu^2} F_{m-2}(\mu); \quad m \geq 0, \quad (31)$$

$$F_0(\mu) = 0,$$

$$F_1(\mu) = \frac{1}{\mu^2} J_0(\mu) - \frac{1}{\mu^2} \int_0^1 J_0(\mu\rho) \, d\rho = J_0(\mu)H_1(\mu) - J_0(\mu)H_0(\mu) \quad (32)$$

and $H_0(\mu)$ and $H_1(\mu)$ are the so-called Struve functions [8].

Finite element solution

Consider the non-linear heat flow equation

$$\rho c \frac{\partial T}{\partial t} = \nabla(\mathbf{K}\nabla T) + q, \quad \nabla = \left[\frac{\partial}{\partial r}, \frac{\partial}{\partial x}\right], \quad (r, x) \in \Omega, \quad (33)$$

where $\mathbf{K}$ is the temperature dependent conductivity tensor, c is the temperature dependent heat capacity and ρ is the density. The region Ω is divided into a number of four-noded isoparametric elements Ω^e, with a quadratic shape function N^i associated with each node i. The unknown function T is approximated through the solution domain at any time t by

$$T = \sum_{i=1}^{n} N^i T^i(t) = \mathbf{NT}, \quad (34)$$

where $\mathbf{T}$ is the column vector of nodal values T^i. The substitution of expansion (34) into equation (33) and the application of the Galerkin method produce the following equation

$$\mathbf{C}\dot{\mathbf{T}} + \mathbf{KT} + F = 0. \quad (35)$$

The form of the matrices **C**, **K** and **F**, together with a description of the temporal discretization of equation (35) and the resulting method of solution of the subsequent equations have been described by many authors [9–12] and will not be considered further.

Experimental studies

For experimental studies, steel rods of the diameter 50 mm have been used. Thermocouples were placed in a non-moving specimen at various distances from the place of contact. The points analysed are shown

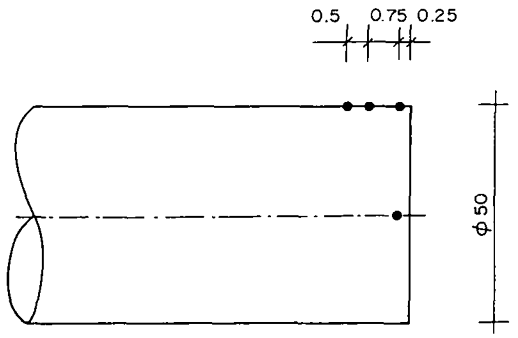

FIG. 2. Thermocouple locations in the rod.

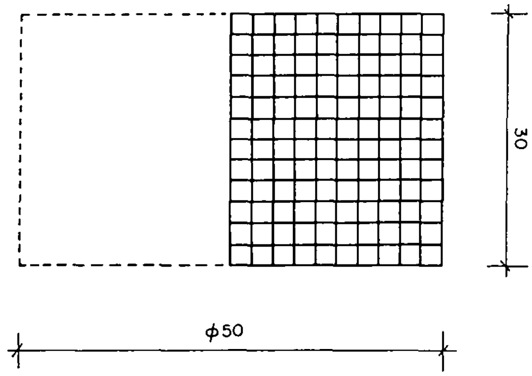

FIG. 3. An assumed four-nodal isoparametric finite element mesh.

in Fig. 2. Each specimen had one thermocouple. Specimens were made of the steel St3 (Polish notation). This steel has the following thermal properties at room temperature: thermal conductivity 42 W m^{-1} K^{-1}, heat capacity 670 J kg^{-1} K^{-1} and density 7800 kg m^{-3}. The influences of temperature on material properties are given in Table 1. The parameters of the process were as follows: heating time 20 s, pressure 20 MN m^{-2}, heat introduced to the surface of contact 84×10^5 W m^{-2}. This heat can be calculated in the following way

Table 1. Influence of temperature on the thermal properties of the steel St3

	Temperature (°C)				
	20	500	800	900	1000
Thermal conductivity [W m^{-1} K^{-1}]	42	50	54	56	56
Heat capacity [J kg^{-1} K^{-1}]	670	1030	1300	680	680

$$q = \int_0^{r_0} p\mu_{fr}\omega r 2\pi r\,dr = \tfrac{2}{3}p\mu_{fr}\omega\pi r_0^3, \qquad (36)$$

where μ_{fr} is the friction coefficient, ω is an angular speed, and p is the pressure.

RESULTS

Because of the considered analytical model of the process, we analyse, in calculations, both the variation of material characteristics with temperature and assume it is independent of temperature. The finite element model of the process is shown in Fig. 3. Results are presented in Figs. 4 and 5. A comparative study indicates quite a good correlation of the three methods considered in this work. Some of the differences were observed on the surface of the specimen. These may be caused by a chosen value of the surface film conductance or the distribution of heat fluxes along the radius and changes of the friction coefficient

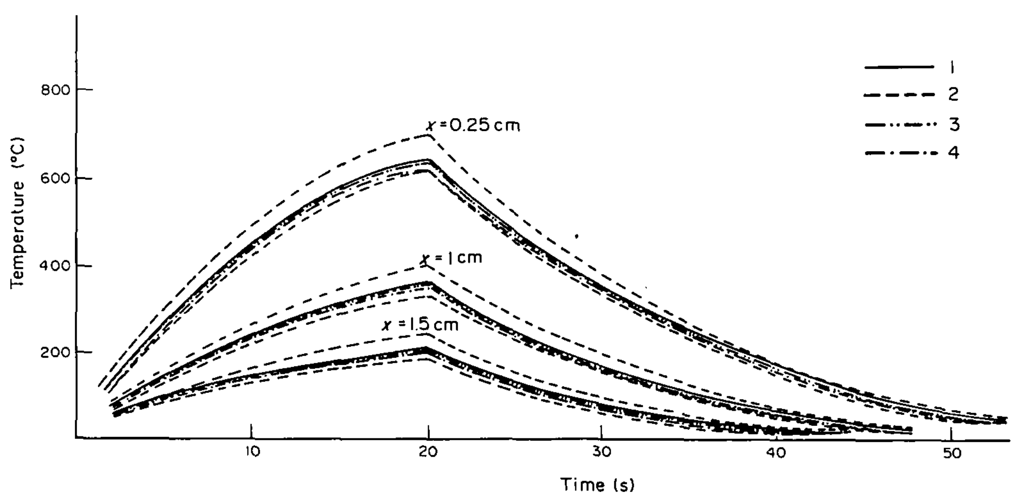

FIG. 4. Temperature as a function of time on the surface of steel friction welded rods: 1, analytical solution; 2, experimental solution; 3, numerical solution if material properties are not a function of temperature; 4, numerical solution if material properties are dependent on temperature.

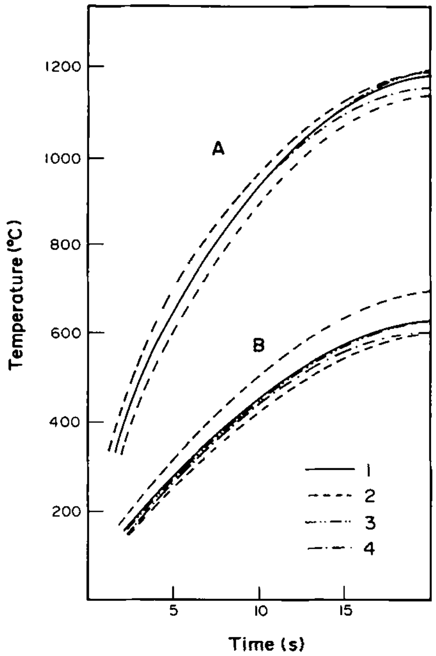

FIG. 5. Temperature as a function of time in friction welded rods: A, on the axis; B, on the surface; curves 1–4, as in Fig. 4.

in the time of the process. Experimental results indicate discrepancy of measurements. This is probably caused by some inaccuracies formed in the phase of preparation of specimens and their fixing in the friction welding device. Small errors can also appear as a result of the thermocouple used and its installation in the specimen.

FINAL REMARKS

The purpose of this paper was to present a comparative investigation of thermal problems in friction welding. The results indicate a good correlation of analytical, numerical and experimental studies.

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